

The Prime-Power Recurrence Principle

A proof for the prime sequence, composite factorization propagation, and a new structure for integer fingerprints

Damon Michael Dorsey

PrimeMind Project

1 September 2026

Abstract

This paper defines the fundamentals of prime and composite integers, and how the principles of factorization describe the position of primes. By identifying the power-factorization of composites, an understanding of the sequence of composites and their primes becomes clear. By defining each integers fundamental factorization, and structuring each factorization into compartments based on their power levels only, a simple formula emerges, that shows the indefinite continuation of a periodic structure. With this understanding, a complete picture of when a prime number will appear in the sequence, without having to know any other previous integers. With this understanding, we then create a coordinate based matrix for this factorization of numbers. This coordinate matrix remains exact throughout all integers, and serves to identify the surrounding integers, composite or prime, by knowing their interior factor-power frequency.

We define a fixed matrix of prime-power coordinates. Column i belongs permanently to the i th prime p_i , the row a belongs permanently to exponent a , and we can then identify every integer with it's coordinate, therefor the cell (i, a) represents the condition $p_i^a \mid n$. It's address never changes. The full matrix's state changes with every integer n , and each cell is exactly periodic with period p_i^a . Higher cells in a prime column are nested inside lower cells, and the finite set of active cells at n is a complete factorization fingerprint. The number of active cells is $\Omega(n)$.

We also distinguish that there is a maximum that can be confined within a single factor layer within this coordinate matrix, before another layer is created. This shows that the factor-power system has an exact staged growth pattern and gives the atlas a mathematically determined outer boundary. This discovery allows a novel form of identification of an integer's general locality, as it's neighborhood in the number sequence can be identified by specific layer maximums in the matrix.

The central structural distinction is that a prime number is not only an isolated integer. Its unseen address continues through every composite containing that same power, which more importantly, continues with exact periodicity indefinitely. Generally speaking, this fingerprint of any integer (m) is contained in the fingerprint of every following composite (n) exactly when $m \mid n$; therefor every composite sub-pattern recurs with exact period m . Prime numbers therefore appear as a new cell in the matrix. Before that cell $(n, 1)$ is introduced, no previously established prime-base coordinate is active at n . This recursive rule generates exactly the prime sequence.

We add two secondary organizations: 'largest-prime-factor branches' with the sharp envelope $p2^{L-1} \leq n \leq p^L$, and 'prime-gap skeletons' that collect the containing composites fingerprints between consecutive prime gaps. A high-scale reduced-coordinate experiment retained all 256 primes in a 5,000-integer window. Finally, we report finite explicit-formula experiments using up to one million tabulated zeta-zero ordinates, where all primes and all zeta-zeros align at a Pearson correlation of approximately 1.0. A local score fixed before two later tests separated all ordinary primes from all non-primes in windows of 500 and 5,000 integers, including higher factorized composites. These results show clear evidence of a general compressed next-prime rule.

Although not proof, these experiments point towards this as being a possible formulation for the prediction of a prime number within the confinements of its known matrix atlas, the previous integers known fingerprint database. This has many implications, not just for number theory and mathematics, but for encryption security and the compression of information.

Contents

1	Purpose, terminology, and claim boundaries	3
2	What a coordinate means	3
2.1	Visual convention for the prime-power factor matrix	4
3	Exact prime-power recurrence	5
4	Integer fingerprints and factor-power levels	6
4.1	The global layer frontier and section compression	7
4.2	Weighted fingerprint identity	8
5	Composite propagation and layer lifting	9
6	Prime boundaries and recursive generation	9
6.1	A complete small example	10
7	A secondary organization: largest-prime-factor branches	11
8	Gap fingerprints and gap skeletons	12
9	High-scale reduced-coordinate experiment	13
10	Analytic bridge through prime-power source events	14
10.1	Finite reconstruction and local score	15
10.2	Frozen threshold tests	16
10.3	What the zeta experiments do and do not show	17
11	From exact representation to possible prediction	17
12	Experimental protocol	18
13	What is established and what remains open	19
14	Questions for mathematical review	19
15	Conclusion	20
A	Notation summary	20
B	Data provenance for the reported numerical results	21

1 Purpose, terminology, and claim boundaries

The aim of this paper is to describe the main formulation so that a mathematician can inspect it without first using the interactive software built specifically for testing these methods. The paper is organized around one exact object: a matrix whose cells are indexed by prime base and exponent. The matrix is static as a coordinate system and dynamic as a multiple integer state system.

Three kinds of statements are kept separate.

Status	Meaning	Examples in this draft
EXACT	Follows from a proof valid for all integers in the stated domain.	Periodicity of each prime-power coordinate; uniqueness of the fingerprint; composite propagation; prime-base birth rule; branch envelope.
EMPIRICAL	Strong verification on named finite data under stated settings.	The reduced-base experiment; finite zeta reconstruction; frozen threshold tests; the prediction of primes without sieves.
OPEN	A research hypothesis, not yet a theorem.	Compression of the complete coordinate state; predictive use of gap skeletons; scale-independent finite-zero classification.

The working names are provisional. We use *Prime-Power Atlas* for the full system, *Prime-Power Factor Matrix* for the fixed coordinate grid, *integer or composite fingerprint* for the active cells at one integer, and *gap skeleton* for the ordered collection of fingerprints between consecutive primes, also referred to as *prime gaps*.

Core claim in plain language. Every prime number and prime power, is a permanent repeating component. It is included and passes through every composite that contains it. An integer is the unique intersection of all such components active at that position. A composite continuously reuses previously established primes or their prime-powers; and a prime is a position at which no earlier prime base or other component is present, so a new base component begins.

The arithmetic content of prime factorizations, valuations, and the von Mangoldt function is classical; see, for example, [Apostol(1976), Hardy and Wright(2008), Montgomery and Vaughan(2007)]. The open question is whether our unified coordinate and prime-to-prime gap representations as unique matrix footprints, expose a useful overall reduction that is not provided in the standard practice.

2 What a coordinate means

Let

$$2 = p_1 < p_2 < p_3 < \cdots, \quad p_1 = 2 < 3 < 5$$

be the primes in increasing order.

Definition 2.1 (Prime-power coordinate). For each prime index $i \geq 1$ and exponent $a \geq 1$, the coordinate address

$$A(i, a)$$

is the permanent cell associated with the prime power and its unique matrix state

$$m_{i,a} = p_i^a.$$

At integer n , its binary activation state is

$$X_{i,a}(n) = \mathbf{1}_{p_i^a | n}. \quad (1)$$

The word *coordinate* refers to the permanent address (i, a) , not to the current integer and not to a geometric claim about physical space. The address specifies which prime family and which exponent level are being measured. The value $X_{i,a}(n)$ is the state observed at n .

A more detailed phase state is

$$r_{i,a}(n) = n \bmod p_i^a, \quad (2)$$

$$\phi_{i,a}(n) = \frac{r_{i,a}(n)}{p_i^a}, \quad (3)$$

$$\delta_{i,a}(n) = (-n) \bmod p_i^a. \quad (4)$$

Here r is the current remainder, ϕ is the normalized position inside one recurrence, and δ is the distance to the next activation. The update is deterministic:

$$r_{i,a}(n+1) = (r_{i,a}(n) + 1) \bmod p_i^a. \quad (5)$$

Therefor cells with different periods advance at different rates, but each cell has a perfectly defined state at every integer.

2.1 Visual convention for the prime-power factor matrix

The first construction adopts the following fixed convention.

1. The origin is the lower-left corner.
2. Prime bases increase from left to right: $2, 3, 5, 7, 11, \dots$
3. Exponent levels increase from bottom to top: $a = 1, 2, 3, \dots$
4. Every cell has the same physical width and height.
5. The matrix grows by adding columns to the right and rows upward. An interactive viewer pans or scrolls; it does not shrink old cells to fit a predetermined frame.

These are representational choices rather than number-theoretic theorems, but they are important because they make comparisons between different integers and different stages of growth visually stable.

Coordinate and state are different objects

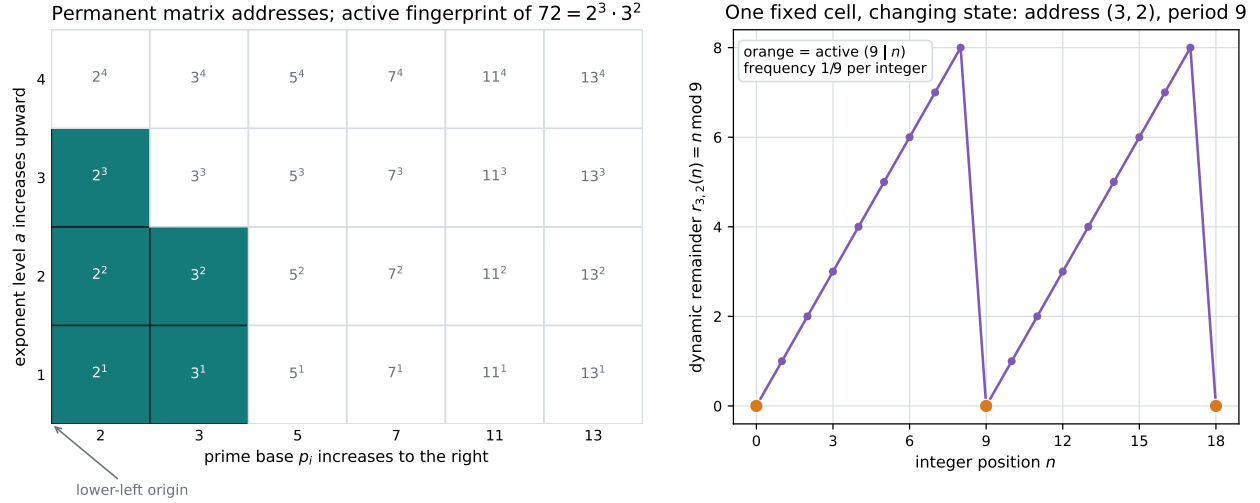


Figure 1: Left: the fixed lower-left coordinate convention. At $n = 72$, the occupied cells are $(2, 1), (2, 2), (2, 3), (3, 1), (3, 2)$. Right: one cell has a permanent address but a changing remainder state, the remainder depicts each next integer. The cell associated with $3^2 = 9$ activates whenever the remainder returns to zero.

3 Exact prime-power recurrence

Theorem 3.1 (Prime-power recurrence). *For every $i \geq 1$ and $a \geq 1$, the activation sequence $X_{i,a}(n)$ has fundamental period p_i^a :*

$$X_{i,a}(n + p_i^a) = X_{i,a}(n) \quad \text{for all integers } n. \quad (6)$$

Proof. The integers n and $n + p_i^a$ have the same remainder modulo p_i^a , so either both are divisible by p_i^a or neither is. To see that no smaller positive period works, note that $X_{i,a}(0) = 1$. If $T > 0$ were a period, then $X_{i,a}(T) = 1$, which requires $p_i^a \mid T$. The smallest such T is p_i^a . $\square$

Corollary 3.2 (Frequency and nesting). *The natural recurrence frequency of cell (i, a) is*

$$\nu_{i,a} = \frac{1}{p_i^a}$$

activations per integer. In addition,

$$X_{i,a+1}(n) \leq X_{i,a}(n) \quad (7)$$

for every n : a higher power can be active only when every lower power in the same prime column is active.

The key point is not merely that the pure number p_i^a exists. The same coordinate is active in every composite containing that power. For example, the cell 2^2 is active at $4, 8, 12, 16, 20, 24, \dots$, even though the other factors differ from one integer to the next.

Remark 3.3 (What is not periodic as one block). The set of all integers satisfying $\Omega(n) = L$ does not have one finite period. Exact recurrence belongs to the individual prime-power coordinates and to specified intersections of those coordinates. This distinction prevents the phrase “the layer repeats” from being interpreted too strongly.

Proposition 3.4 (Finite atlas recurrence). *For a finite set E of coordinate addresses, the joint state vector*

$$(X_{i,a}(n))_{(i,a) \in E}$$

repeats with period

$$T_E = \text{lcm}\{p_i^a : (i, a) \in E\}.$$

If E contains, for each retained prime p_i , all exponents through a_i , then

$$T_E = \prod_i p_i^{a_i}.$$

The infinite atlas has no single finite global period because new primes and higher exponents keep entering. Every finite stage, however, is exactly recurrent.

4 Integer fingerprints and factor-power levels

Definition 4.1 (Prime-Power Factor Matrix and fingerprint). For an integer $n \geq 2$, its Prime-Power Factor Matrix is the infinite binary array

$$M(n) = [X_{i,a}(n)]_{i,a \geq 1}.$$

Its finite active support

$$\mathcal{F}(n) = \{(i, a) : X_{i,a}(n) = 1\} \tag{8}$$

is called the prime-power fingerprint of n .

Although the matrix is infinite, every integer has only finitely many active cells.

Theorem 4.2 (Fingerprint reconstruction). *For each prime p_i ,*

$$v_{p_i}(n) = \sum_{a \geq 1} X_{i,a}(n). \tag{9}$$

Consequently,

$$n = \prod_i p_i^{\sum_{a \geq 1} X_{i,a}(n)}, \tag{10}$$

and $\mathcal{F}(n)$ uniquely determines n .

Proof. If $v_{p_i}(n) = e$, then $p_i^a \mid n$ exactly for $a = 1, \dots, e$. The sum in [equation \(9\)](#) is therefore e . Substituting the recovered exponents into the unique prime factorization gives [equation \(10\)](#). $\square$

Corollary 4.3 (Global factor-power level). *The number of active cells is*

$$|\mathcal{F}(n)| = \sum_i \sum_{a \geq 1} X_{i,a}(n) = \Omega(n). \tag{11}$$

Thus the total factor-power level is the total number of active nested coordinates.

This level groups together different shapes. For example, all of

$$8 = 2^3, \quad 12 = 2^2 \cdot 3, \quad 18 = 2 \cdot 3^2, \quad 27 = 3^3, \quad 30 = 2 \cdot 3 \cdot 5$$

have $\Omega(n) = 3$, but their occupied columns differ.

4.1 The global layer frontier and section compression

The total factor-power level in Equation (11) also determines an exact upper frontier for the growing matrix. This frontier does not describe the level of every individual integer. Instead, it describes the greatest factor-power level that can have appeared by a given endpoint.

Proposition 4.4 (Global layer-frontier law). *For every integer $N \geq 2$,*

$$L_{\max}(N) := \max_{2 \leq n \leq N} \Omega(n) = \lfloor \log_2 N \rfloor. \quad (12)$$

Equivalently, factor-power level L first becomes possible at the integer

$$n = 2^L. \quad (13)$$

Consequently, for every endpoint satisfying

$$2^L \leq N < 2^{L+1}, \quad (14)$$

the running maximum factor-power level through N is exactly L .

Proof. If $\Omega(n) = L$, then n is a product of L prime factors, counted with repetition. Every prime factor is at least 2, so

$$n \geq 2^L.$$

Therefore no integer below 2^L can occupy factor-power level L .

Conversely,

$$2^L = \underbrace{2 \cdot 2 \cdots 2}_{L \text{ factors}}$$

has $\Omega(2^L) = L$. Thus level L first appears exactly at 2^L , and the greatest level attained through N is $\lfloor \log_2 N \rfloor$. $\square$

The running layer frontier therefore rises in the exact staircase

$$2, 4, 8, 16, 32, 64, \dots,$$

with one new maximum level introduced at each power of 2. For example, every endpoint from 32 through 63 has running maximum level 5, while every endpoint from 64 through 127 has running maximum level 6. Individual integers inside either interval may lie on much lower levels.

Corollary 4.5 (Finite matrix height). *A Prime-Power Factor Matrix constructed through endpoint N requires no exponent row above*

$$\lfloor \log_2 N \rfloor.$$

More specifically, the greatest possible occupied exponent in the column belonging to prime p_i is

$$a_{\max}(p_i; N) = \lfloor \log_{p_i} N \rfloor \leq \lfloor \log_2 N \rfloor. \quad (15)$$

This produces a natural partition of the integer sequence into *layer regimes*

$$\mathcal{R}_L := \{n \in \mathbb{Z} : 2^L \leq n < 2^{L+1}\}. \quad (16)$$

All endpoints within the same regime share the same global matrix ceiling. This gives sections of the number sequence a common structural scale before their finer coordinate patterns are compared.

Exact procedural compression. Let E be any finite set of retained coordinate addresses and let $I = [A, B]$ be a finite integer interval. Define the left-boundary phase state

$$\Sigma_E(A) := (r_{i,a}(A))_{(i,a) \in E}. \quad (17)$$

Because every coordinate evolves according to

$$r_{i,a}(A + t) = (r_{i,a}(A) + t) \bmod p_i^a, \quad (18)$$

the complete activation pattern of all coordinates in E throughout I can be regenerated from:

1. the retained coordinate set E ;
2. the initial state $\Sigma_E(A)$;
3. the interval length $B - A$.

Thus a finite section need not be stored as a separate value for every cell at every integer. It admits an exact, lossless generating description consisting of a left-boundary state and the permanent recurrence rules. The layer-frontier law adds a canonical vertical boundary to that description.

Open structural-compression question. The layer regimes also suggest a more ambitious form of compression. A numerical section might be summarized by a smaller signature containing its layer regime, coordinate births, left-boundary phase state, level-occupancy profile, and completed gap skeletons. Such a signature could make structurally related sections easier to identify and compare.

Whether a substantially reduced signature retains enough information to locate or rank future prime boundaries is not established by the layer-frontier law. The exact result supplies the hierarchy and the lossless generating description; the predictive reduction remains an open experimental question.

4.2 Weighted fingerprint identity

Let Λ be the von Mangoldt function:

$$\Lambda(d) = \begin{cases} \log p, & d = p^a \text{ for some prime } p \text{ and } a \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\log n = \sum_i v_{p_i}(n) \log p_i = \sum_{(i,a) \in \mathcal{F}(n)} \log p_i = \sum_{d|n} \Lambda(d). \quad (19)$$

For $72 = 2^3 \cdot 3^2$, the active cells are $2, 2^2, 2^3, 3, 3^2$, so the weighted sum is

$$3 \log 2 + 2 \log 3 = \log 72.$$

Equation (19) is the exact analytic bridge between prime-power source events and their propagation through all composites containing them.

5 Composite propagation and layer lifting

The preceding section describes a single integer. The stronger recurrence statement concerns any composite pattern reused inside later integers.

Definition 5.1 (Composite subpattern indicator). For $m \geq 2$, define

$$P_m(n) = \prod_{(i,a) \in \mathcal{F}(m)} X_{i,a}(n). \quad (20)$$

Theorem 5.2 (Universal composite propagation). For all integers $m, n \geq 2$,

$$P_m(n) = 1 \iff \mathcal{F}(m) \subseteq \mathcal{F}(n) \iff m \mid n. \quad (21)$$

Therefore the required fingerprint of m reappears inside every multiple of m , and the sequence $P_m(n)$ has fundamental period m .

Proof. Write $m = \prod_i p_i^{e_i}$. The product in equation (20) equals one exactly when $p_i^a \mid n$ for every i and every $1 \leq a \leq e_i$. This is equivalent to $p_i^{e_i} \mid n$ for all i , hence to $m \mid n$. The active set of P_m is therefore the set of multiples of m , whose fundamental period is m . $\square$

This theorem formalizes the main structural distinction of the project. The fingerprint of $12 = 2^2 \cdot 3$ is present at 12, 24, 36, 48, 60, $\dots$. Those later integers may contain additional active cells, but the three required coordinates $2, 2^2, 3$ remain present as a recurring substructure.

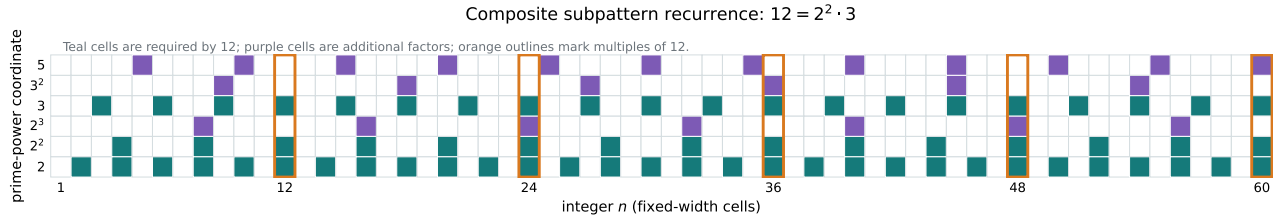


Figure 2: A composite fingerprint repeats as a contained subpattern. The teal cells required by 12 recur at every multiple of 12; purple cells show additional factors that do not erase the recurring 12 pattern. The integer axis uses fixed-width cells.

Corollary 5.3 (Layer lifting). For every prime p , exponent $a \geq 1$, and integer $m \geq 1$,

$$\Omega(p^a m) = \Omega(m) + a. \quad (22)$$

In the matrix, multiplication by p^a raises the height of the p column by exactly a , whether or not p already divides m .

The atlas therefore connects levels by exact multiplicative lifts. Every established fingerprint can be transferred to a higher factor-power level by attaching a prime power.

6 Prime boundaries and recursive generation

A prime has a full fingerprint after it is recognized: its own base cell $(p, 1)$ is active. The generative event occurs immediately before that cell is introduced.

Definition 6.1 (Inherited base support). Suppose all prime bases less than n have already been introduced. Define

$$I(n) = \sum_{\substack{p < n \\ p \text{ prime}}} \mathbf{1}_{p|n}. \quad (23)$$

This counts previously established base coordinates active at n .

Theorem 6.2 (Prime-boundary characterization). *For every $n \geq 2$,*

$$n \text{ is prime} \iff I(n) = 0. \quad (24)$$

Equivalently, it suffices to inspect prime bases $p \leq \sqrt{n}$.

Proof. A prime has no proper prime divisor, so no previously established base is active. A composite n has a prime divisor $p \leq \sqrt{n} < n$, so at least one earlier base is active. $\square$

Corollary 6.3 (Recursive atlas generation). *Start at $n = 2$ with no base columns. Move through the integers in order. Whenever $I(n) = 0$, introduce a new base column labeled n . The introduced labels are exactly the primes, in increasing order.*

This is the sense in which the atlas builds the prime sequence internally. A composite is formed from established coordinates; a prime is a new-base event. The statement is exact, but it is not by itself a claim of lower computational complexity than ordinary divisibility testing.

6.1 A complete small example

From 19 to 23:

$$20 = 2^2 \cdot 5, \quad 21 = 3 \cdot 7, \quad 22 = 2 \cdot 11.$$

Each interior integer is occupied by inherited coordinates. At 23, none of the earlier base coordinates is active, and the new base 23 is introduced.

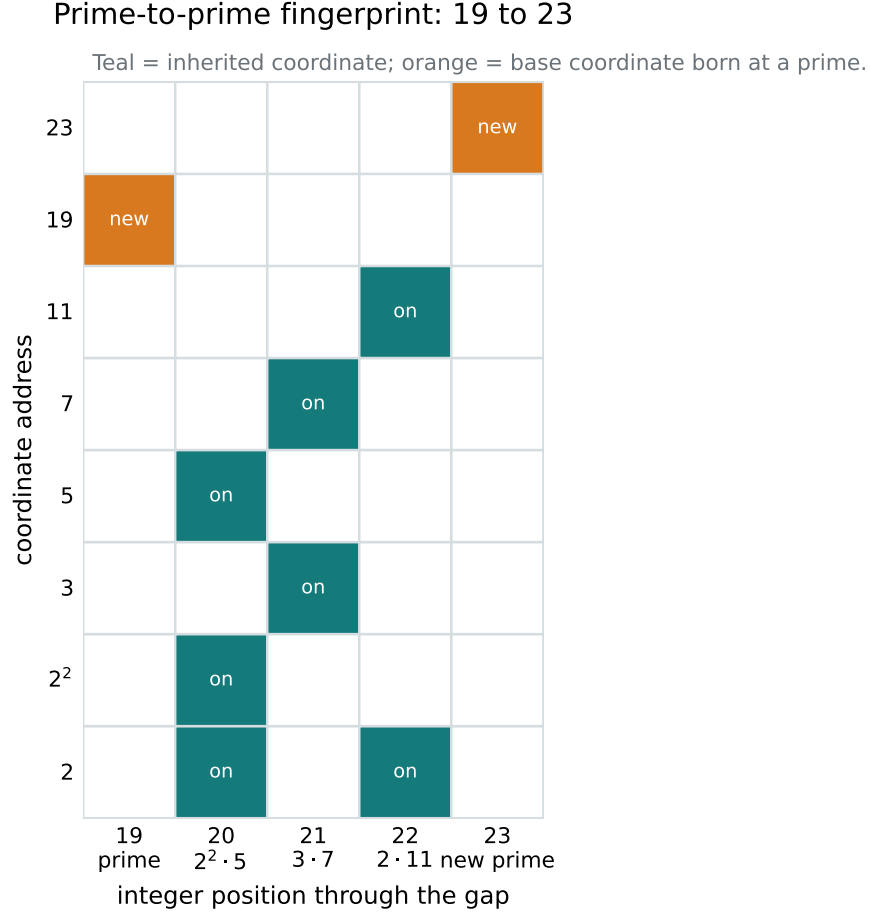


Figure 3: An exact prime-to-prime example. Teal cells are inherited factor coordinates. Orange cells mark the base coordinates introduced at the two boundary primes.

For a bounded forward interval ending at N , the complete base set $p \leq \sqrt{N}$ identifies every prime in that interval exactly. Higher exponent rows enrich the composite fingerprint and factor-power level, but the base row already determines whether any retained prime family divides the candidate.

7 A secondary organization: largest-prime-factor branches

The matrix answers which prime powers are present. A complementary organization assigns each integer to the largest prime in its factorization.

Definition 7.1 (Largest-prime-factor branch). For a prime p and level $L \geq 1$, let

$$\mathcal{B}_{p,L} = \{n : \text{LPF}(n) = p, \Omega(n) = L\}.$$

Theorem 7.2 (Branch-layer envelope). Every $n \in \mathcal{B}_{p,L}$ satisfies

$$p2^{L-1} \leq n \leq p^L, \tag{25}$$

and both bounds are attained.

Proof. Remove one occurrence of the largest prime factor p . The remaining $L - 1$ prime factors are each at least 2 and at most p . Their product therefore lies between 2^{L-1} and p^{L-1} . Multiplying by p gives the bounds. Equality occurs at $p2^{L-1}$ and p^L . $\square$

On logarithmic axes, the bounds are straight:

$$\log p + (L - 1) \log 2 \leq \log n \leq L \log p.$$

This does not imply that every integer between the boundaries belongs to the branch; the envelope is attainable but sparsely occupied.

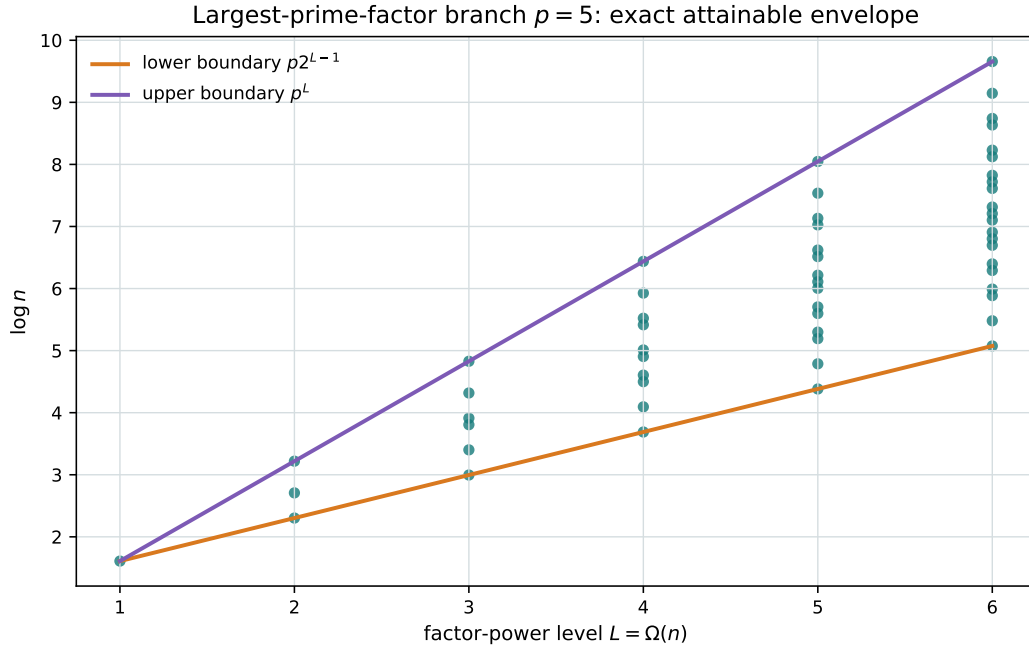


Figure 4: All members of the $p = 5$ branch through factor-power level 6 lie between the exact lower and upper boundaries. The line shape is caused by taking logarithms; the occupied points are the arithmetic content.

This branch view is useful for organizing completed composite fingerprints and for defining a gap skeleton without discarding factor ownership.

8 Gap fingerprints and gap skeletons

Let $P < Q$ be consecutive primes and $g = Q - P$. The full coordinate record of the gap is the array

$$G_{i,a,t}(P, Q) = X_{i,a}(P + t), \quad 0 \leq t \leq g. \quad (26)$$

Each time slice t is one integer fingerprint. A two-dimensional display can place integer offset horizontally and prime-power address vertically. A linked geometric rendering can add factor heights, largest-prime-factor ownership, and repeated factor occurrences; this is the current working meaning of *gap skeleton*.

Two uses must be distinguished.

Completed gap skeleton. Built after the right boundary Q and all interior factorizations are known. It is descriptive truth and can be used to search for recurring structures in earlier gaps.

Causal gap prefix. Built from the known left boundary P and updated one integer at a time, without using the future right boundary. It is admissible in a prospective prediction test.

The completed skeleton should not be discarded merely because it is post-reveal information. It may suggest a smaller lawful summary of past gaps, a classification of residual composites, or a more efficient organization of the coordinate state. Those possibilities are empirical questions.

A natural causal state at the left prime is

$$\Sigma_E(P) = (r_{i,a}(P))_{(i,a) \in E}, \quad (27)$$

for a chosen finite coordinate set E . Equation (4) determines the entire future rollout of that selected state. The research problem is whether a much smaller object than the complete state can preserve the location of the next new-base event.

Working conjecture 8.1 (Gap-state compression). *There may exist a reduced statistic of the current coordinate state, past completed gap skeletons, or both, that ranks the next prime boundary better than an equally costly reduced-base screen.*

No universal form of this statistic has yet been established.

9 High-scale reduced-coordinate experiment

A research export was generated from

$$S = 459,824,117, \quad H = 5,000,$$

covering the future integers through 459,829,117. The complete base set through $\sqrt{N}$ contained 2,406 primes and left exactly the 256 primes in the window. A reduced set containing only the 25 prime bases through 97 left 600 candidates:

256 primes + 344 unresolved composites.

It therefore eliminated 4,400 of 4,744 composites, or 92.75%, while retaining every prime. The pre-reveal coordinate fields and post-reveal truth were exported separately and independently audited in the project record [PrimeMind project(2026)].

Table 1: Candidate reduction in the 5,000-integer high-scale window.

Retained base set	Base count	Survivors	True primes	False survivors
First 10 bases, through 29	10	788	256	532
Through 31	11	762	256	506
Through 97	25	600	256	344
Complete through $\sqrt{N}$	2,406	256	256	0

Across 256 completed prime gaps, the next prime was the first reduced-set survivor in 42.58% of the gaps, within the first two in 67.97%, and within the first five in 93.75%. Those values closely matched

the baseline implied by the 256/600 prime fraction among all survivors. Thus the reduced set is a strong compressor, but chronological survivor order did not display an additional predictive law.

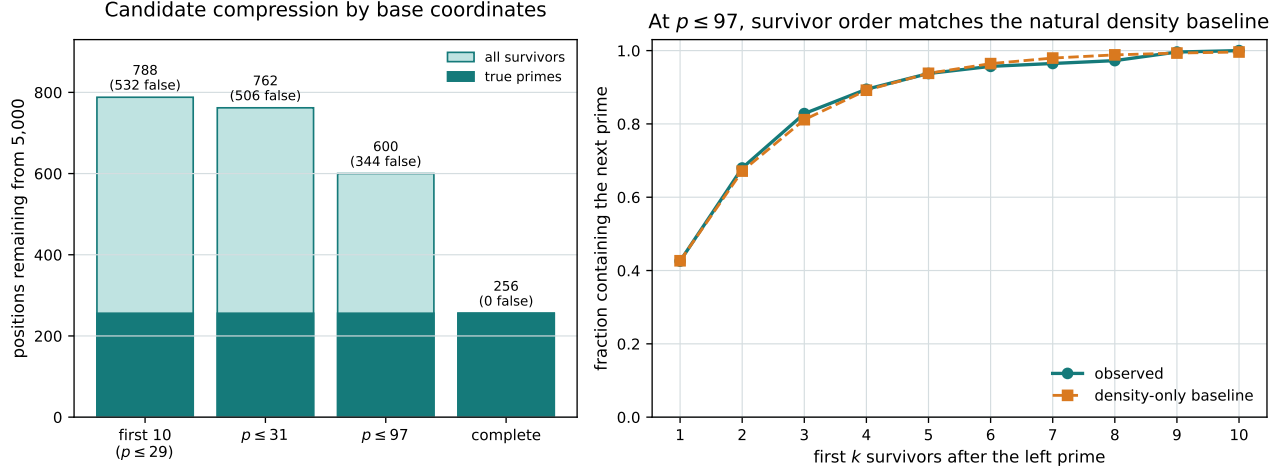


Figure 5: Left: candidate reduction as more base coordinates are retained. Right: for the bases through 97, the observed next-prime rank is close to the density-only baseline.

A further exact limitation was exposed. Any candidate surviving every retained base coordinate also survives all higher powers belonging to those same bases. Therefore the candidate’s local higher-power values are all zero. Predictive information from higher rows, if it exists, must come from the neighboring pattern, the approach to the candidate, completed past gaps, or a separate analytic signal rather than from the candidate’s own retained higher-power activations.

10 Analytic bridge through prime-power source events

The atlas propagates prime-power coordinates through all their multiples. Analytic prime theory also singles out prime powers, but first as source events. Define

$$\psi(x) = \sum_{n \leq x} \Lambda(n).$$

The classical explicit formula relates the fluctuation of $\psi(x)$ to the nontrivial zeros ρ of the Riemann zeta function. In one standard midpoint convention,

$$\psi_0(x) = x - \sum_{\rho} \frac{x^{\rho}}{\rho} - \log(2\pi) - \frac{1}{2} \log(1 - x^{-2}), \quad (28)$$

with the usual interpretation at jump points; see [Titchmarsh(1986), NIST DLMF].

The conceptual link to the atlas is equation (19): the same prime-power sources that contribute to Λ are repeated through composite fingerprints by divisibility. In Dirichlet-series form,

$$\zeta(s) \left(-\frac{\zeta'(s)}{\zeta(s)} \right) = -\zeta'(s), \quad (29)$$

which mirrors “source events times propagation through multiples equals the weighted logarithmic structure of all integers.”

This identity does not make the zeta zeros a proof of the coordinate atlas; the elementary atlas theorems already have direct proofs. It does show that prime powers are the correct shared arithmetic object.

10.1 Finite reconstruction and local score

Let $E_K(n)$ denote the anchored residual reconstructed from the first K positive zero ordinates and their conjugates, with the correction terms used by the experiment. A local prime-power estimate is obtained by first differences:

$$\hat{\Lambda}_K(n) = 1 + E_K(n) - E_K(n-1). \quad (30)$$

The normalized score is

$$Z_K(n) = \frac{\hat{\Lambda}_K(n)}{\log n}. \quad (31)$$

Ideally, an ordinary prime has score near 1, a prime square near $1/2$, a prime cube near $1/3$, and an integer that is not a prime power near 0.

On the same 500-integer window beginning at 61,729, increasing K from 500,000 to 1,000,000 improved both cumulative and local agreement.

Table 2: Same-window finite-zero convergence near 61,729.

Metric	$K = 500,000$	$K = 1,000,000$
Cumulative correlation	0.999762	0.999980
Mean absolute cumulative error	0.803	0.098
Maximum cumulative error	1.932	0.380
Local jump correlation	0.993805	0.997894
Mean absolute local jump error	0.382	0.162

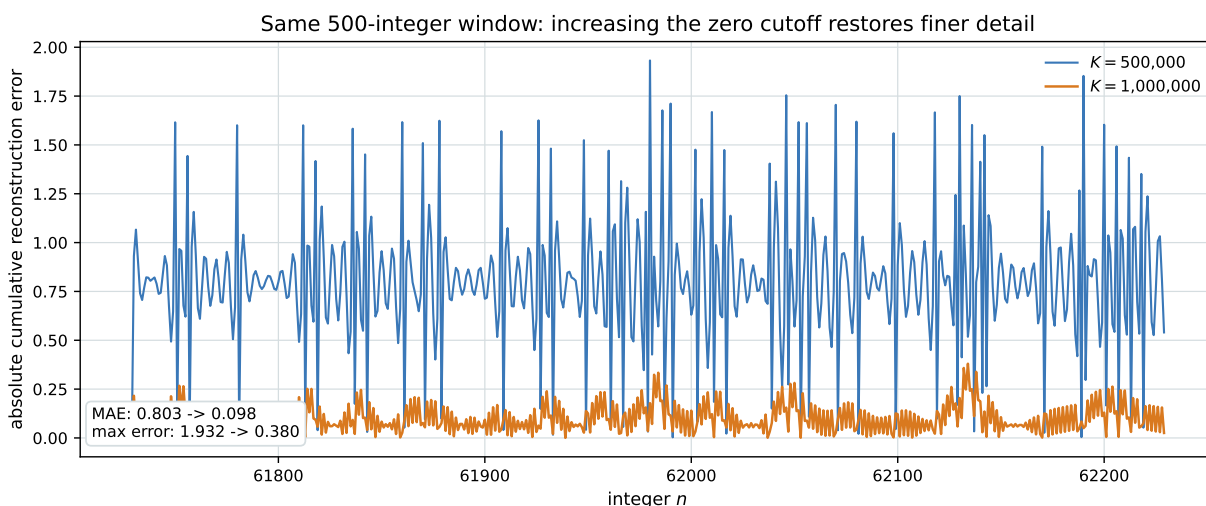


Figure 6: Absolute cumulative reconstruction error on an identical 500-integer window. The added zero ordinates supply substantially finer local information at this scale.

10.2 Frozen threshold tests

Exploratory calculations suggested the rule

$$Z_{1,000,000}(n) > 0.75 \implies \text{classify } n \text{ as an ordinary prime.} \quad (32)$$

The threshold was then fixed before two later runs.

Table 3: Prospective finite-zero tests of the fixed threshold in [equation \(32\)](#).

Window	Integers	Primes	Nonprimes	Errors	Additional stress
70,002–70,501	500	50	450	0	No higher prime powers in the interval.
72,020–77,019	5,000	440	4,560	0	Three prime squares were correctly kept below the threshold.

In the 5,000-integer stress run, ordinary-prime scores ranged from 1.0731 to 1.1567. The largest nonprime score was 0.5722, attained at $277^2 = 76,729$. The other prime squares were $269^2 = 72,361$ with score 0.5571 and $271^2 = 73,441$ with score 0.5517. A phase-randomized comparison using the same frequencies did not reproduce the result: it selected only 4 of 440 primes and produced 32 false positives.

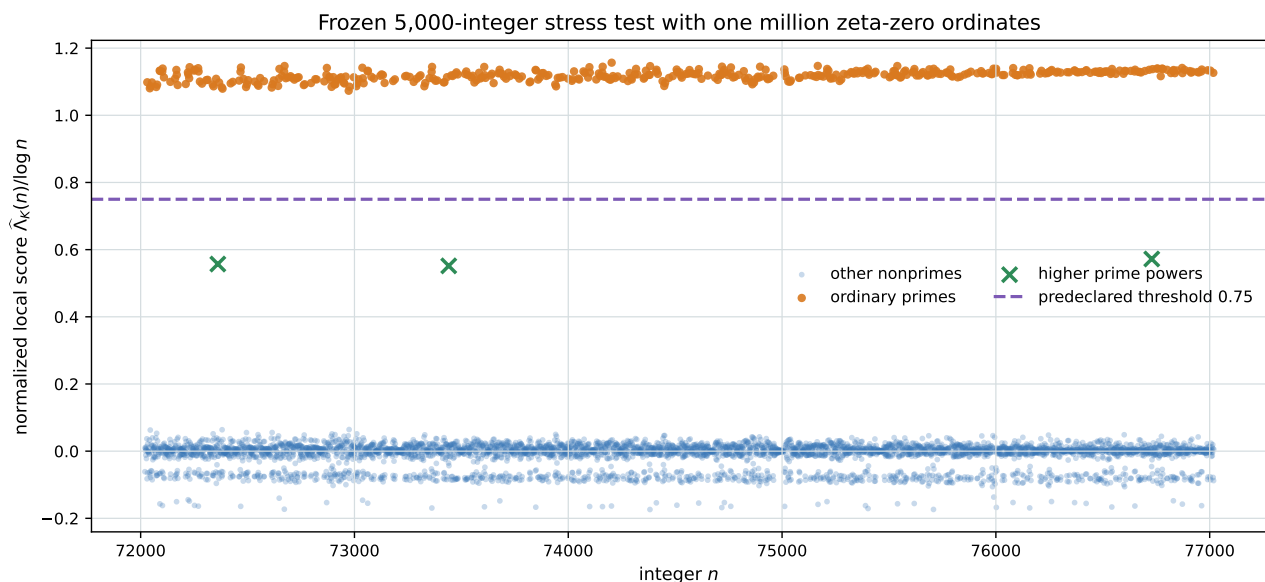


Figure 7: Normalized local scores in the 5,000-integer frozen stress test. Ordinary primes, higher prime powers, and other nonprimes form visibly distinct bands in this finite window.

The cumulative curves over the same interval had correlation 0.999900, while the local event correlation was 0.994036. The local phase-randomized correlation was 0.010586.

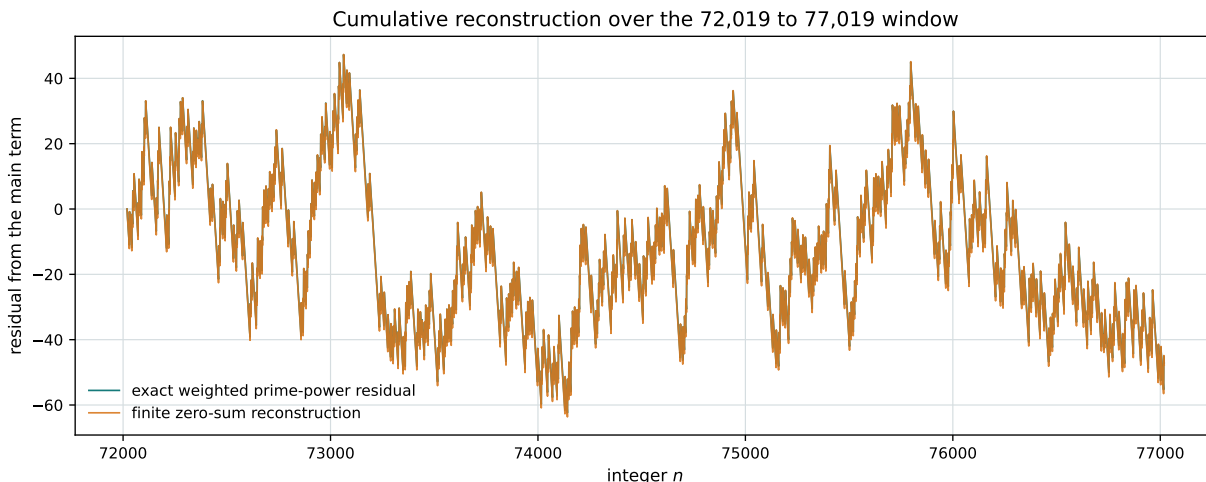


Figure 8: The exact weighted prime-power residual and the finite reconstruction over the stress interval. The local first differences, rather than visual overlap alone, are the classification-relevant quantity.

10.3 What the zeta experiments do and do not show

The data support the following finite statement: at the tested scales and zero cutoffs, the truncated explicit-formula calculation recovered enough local prime-power structure for a predeclared threshold to separate the tested ordinary primes from nonprimes. They do not yet prove that the threshold works for every integer, that one million zeros are sufficient at all scales, or that this method is computationally preferable to standard primality procedures. They also do not prove the Riemann hypothesis; only a finite list of tabulated zeros was used [Odlyzko].

11 From exact representation to possible prediction

The exact framework already gives a next-prime construction when all necessary base coordinates are retained. Starting from a known prime P , advance the recurrence states and choose the first later integer at which no required earlier base coordinate is active. The unresolved research question is whether the complete information can be reduced.

The following information sources should be tested separately and in combination.

1. **Reduced base state.** Retain only a finite initial set of prime bases and measure the unresolved survivors.
2. **Power-neighborhood state.** Use higher-row activations near, but not at, each survivor; candidate-local values alone cannot help after the corresponding base rows have missed.
3. **Completed past gap skeletons.** Search for stable summaries or recurring substructures without using the unknown right endpoint of the current gap.
4. **Branch residuals.** Remove geometry forced by the branch envelope and test whether residual occupancy or spacing transfers across scale.
5. **Finite spectral score.** Use a fixed $Z_K(n)$ rule as a secondary ranker, with zero cutoff and threshold frozen before the target interval is opened.

Working conjecture 11.1 (Prime-Power State Compression). *There exists a family of reduced states, substantially smaller than the complete set of required base coordinates, that identifies or ranks the next prime boundary with reproducible advantage over a matched arithmetic baseline.*

At present this is a conjecture. The high-scale reduced-base experiment shows strong compression but no extra chronological ranking; the finite-zero tests show local classification at two tested windows but do not establish universal or efficient behavior.

12 Experimental protocol

A credible prediction claim requires a strict information boundary.

Freeze

Before examining a target interval, record:

- the starting prime and horizon;
- the retained coordinate set;
- all phase or remainder states at the starting point;
- any past-gap training range;
- all formulas, thresholds, zero cutoffs, and random seeds;
- the computational budget and comparison method.

Predict

Write a locked list of candidate scores and the predicted next prime before target factorizations or future gap boundaries are used.

Reveal

Only afterward compute target primality, factorization, Ω , LPF, and completed gap skeletons. Report every interval, including failures.

Controls

At minimum compare against:

- the same reduced base set without extra features;
- chronological survivor order;
- phase-randomized spectral terms;
- shifted or permuted past-gap summaries;

- equal-cost arithmetic baselines;
- disjoint windows at increasing numerical scales.

The primary outputs should include exact next-prime success, true-prime rank, top- k coverage, false positives, missed primes, operation count, and performance as numerical scale increases.

13 What is established and what remains open

Status	Statement
EXACT	Every cell (i, a) is periodic with period p_i^a and frequency $1/p_i^a$.
EXACT	The active cells uniquely reconstruct the integer and their number is $\Omega(n)$.
EXACT	The fingerprint of m is contained in that of n exactly when $m \mid n$; every composite subpattern therefore recurs.
EXACT	Multiplication by p^a raises total factor-power level by exactly a .
EXACT	A new base is introduced exactly at a prime; complete required base coordinates identify primes in a bounded interval.
EXACT	Largest-prime-factor branch members at level L lie sharply between $p2^{L-1}$ and p^L .
EMPIRICAL	The 25 bases through 97 eliminated 92.75% of composites in one 5,000-integer high-scale window while retaining every prime.
EMPIRICAL	A finite one-million-zero local score with a frozen threshold passed two later windows totaling 5,500 integers.
OPEN	A compressed gap or recurrence fingerprint always predicts the next prime.
OPEN	Completed gap skeletons yield a general reduction in computational work.
OPEN	The finite spectral threshold remains valid at arbitrary scale or supplies a practical prime algorithm.
OPEN	Any part of the elementary theorem package is historically new rather than a new organization of known facts.

14 Questions for mathematical review

This draft is intended to make criticism easy. The most useful questions for an external reader are:

1. Is the distinction between a permanent coordinate address and its changing state mathematically unambiguous?
2. Is the Prime-Power Factor Matrix best treated as a valuation diagram, an incidence matrix, or another existing standard object?
3. Is the composite propagation theorem in [equation \(21\)](#) the clearest formal statement of the project's central distinction?
4. Which theorem names should be replaced by standard terminology?

5. Is the recursive new-base rule best presented as a characterization, an algorithm, or an expository reformulation?
6. Does the gap array in [equation \(26\)](#) suggest known symbolic-dynamics, automata, or valuation-state machinery that should be cited?
7. Are the finite explicit-formula normalization and first-difference score stated with enough precision for independent reproduction?
8. What is the fairest complexity baseline for a future compression claim?
9. Which part of the integrated architecture, if any, appears genuinely new enough to justify a separate research contribution?

15 Conclusion

The foundational structure can be stated compactly:

Every prime-power coordinate is permanent and exactly recurrent. Those coordinates pass through all composites containing them. Their intersections form unique integer fingerprints; their total occupancy gives the factor-power level; and a prime is the point at which no previously established prime base is active, so a new base begins.

This is sufficient to define a coherent recurrence atlas and to generate the prime sequence exactly. It does not yet establish that the complete state can be replaced by a smaller universal predictor. The strongest next stage is therefore not to add more visual complexity indiscriminately, but to use the fixed atlas, completed past gap skeletons, and strict prospective tests to determine which information is genuinely compressible.

A Notation summary

Symbol	Meaning
p_i	The i th prime.
$A(i, a)$	Permanent matrix address: prime index i , exponent level a .
$X_{i,a}(n)$	Binary activation of cell (i, a) at integer n ; one exactly when $p_i^a \mid n$.
$r_{i,a}(n)$	Remainder of n modulo p_i^a .
$\phi_{i,a}(n)$	Normalized recurrence position $r_{i,a}(n)/p_i^a$.
$\delta_{i,a}(n)$	Distance from n to the next multiple of p_i^a .
$M(n)$	Infinite Prime-Power Factor Matrix state at integer n .
$\mathcal{F}(n)$	Finite set of active cells; the integer fingerprint.
$v_p(n)$	Exponent of prime p in n .
$\Omega(n)$	Total number of prime factors with multiplicity; equal to $ \mathcal{F}(n) $.

Symbol	Meaning
$P_m(n)$	Indicator that the entire fingerprint of m is present inside n ; equal to $\mathbf{1}_{m n}$.
$I(n)$	Number of previously established prime bases dividing n .
$\text{LPF}(n)$	Largest prime factor of n .
$G_{i,a,t}(P, Q)$	Prime-power activation at offset t inside the completed gap from P to Q .
$\Lambda(n)$	Von Mangoldt weight: $\log p$ at p^a , zero otherwise.
$\psi(x)$	Cumulative weighted prime-power count.
$\hat{\Lambda}_K(n)$	Local prime-power estimate obtained from a finite zero-sum first difference.
$Z_K(n)$	Normalized local score $\hat{\Lambda}_K(n)/\log n$.

B Data provenance for the reported numerical results

The figures and tables in the empirical sections were generated from the following internal exports:

- `prime-research-package-S459824117-H5000-cutoff (1).json`;
- `prime-research-package-S459824117-H5000-full.json`;
- `prime-zeta-comparison-S61729-H5000-K500000.csv`;
- `prime-zeta-comparison-S61729-H500-K1000000.csv`;
- `prime-zeta-comparison-S70001-H500-K1000000.csv`;
- `prime-zeta-comparison-S72019-H5000-K1000000.csv`.

The Overleaf bundle accompanying this includes the derived metrics, figure-generation script, and source checksums. The full raw exports are retained separately because some exceed convenient manuscript-bundle size.

References

- [Apostol(1976)] Tom M. Apostol. *Introduction to Analytic Number Theory*. Springer-Verlag, 1976.
- [Hardy and Wright(2008)] G. H. Hardy and E. M. Wright. *An Introduction to the Theory of Numbers*, 6th edition. Oxford University Press, 2008.
- [Montgomery and Vaughan(2007)] Hugh L. Montgomery and Robert C. Vaughan. *Multiplicative Number Theory I: Classical Theory*. Cambridge University Press, 2007.
- [Titchmarsh(1986)] E. C. Titchmarsh. *The Theory of the Riemann Zeta-Function*, 2nd edition, revised by D. R. Heath-Brown. Oxford University Press, 1986.
- [NIST DLMF] NIST Digital Library of Mathematical Functions. Section 25.16, Mathematical Applications. <https://dlmf.nist.gov/25.16>.

- [Odlyzko] Andrew M. Odlyzko. Tables of zeros of the Riemann zeta function. https://www-users.cse.umn.edu/~odlyzko/zeta_tables/index.html.
- [PrimeMind project(2026)] Damon Michael Dorsey. *PrimeMind Research Exports and Core Technical Recorder*. Unpublished computational datasets and internal project documentation, September 2026.